Trigonometric Equations

Question1

$1 + \cos x + \cos^2 x + \cos^3 x + \ldots$ **to** $\infty = 4 + 2\sqrt{3}$**, then** $x =$

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Options:

A.

$\frac{n\pi}{6}$

B.

$(4n \pm 1)\frac{\pi}{3}$

C.

$(12n \pm 1)\frac{\pi}{6}$

D.

$(3n \pm 1)\frac{\pi}{3}$

Answer: C

Solution:

Given equation is

$$1+\cos x+\cos^2 x+\ldots+\infty=4+2\sqrt{3}$$
$$\Rightarrow \frac{1}{1-\cos x}=4+2\sqrt{3}$$
$$\Rightarrow 1-\cos x=\frac{1}{4+2\sqrt{3}}\times\frac{4-2\sqrt{3}}{4-2\sqrt{3}}$$
$$\Rightarrow 1-\cos x=\frac{4-2\sqrt{3}}{4}\Rightarrow 1-\cos x=1-\frac{\sqrt{3}}{2}$$
$$\Rightarrow \cos x=\frac{\sqrt{3}}{2}=\cos\frac{\pi}{6}$$
$$\Rightarrow x=2n\pi\pm\frac{\pi}{6}\Rightarrow x=\frac{12n\pi\pm\pi}{6}$$
$$\Rightarrow x=(12n\pm 1)\frac{\pi}{6}$$

Question2

α, β are the roots of the equation $\sin^2 x+b\sin x+c=0$. If $\alpha+\beta=\frac{\pi}{2}$, then $b^2-1=$

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Options:

A. C

B. $2c$

C. C^2

D. $4c^2$

Answer: B

Solution:

Given, $\sin^2 x+b\sin x+c=0$ and

$\alpha+\beta=\frac{\pi}{2}$

Let $y=\sin x$, thus we get

$y^2+by+c=0$

The roots of this equation are $\sin\alpha$ and $\sin\beta$.

So, sum of the roots $=\sin\alpha+\sin\beta=-\frac{b}{1}$

Product of the roots $=\sin\alpha\cdot\sin\beta=c$

Since, $\alpha+\beta=\frac{\pi}{2}$

$\Rightarrow \quad \beta = \frac{\pi}{2} - \alpha$

So, $\sin\alpha + \sin\left(\frac{\pi}{2} - \alpha\right) = -b$

$\Rightarrow \sin\alpha + \cos\alpha = -b$

$$\left[\because \sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta\right]$$

$\Rightarrow (\sin\alpha + \cos\alpha)^2 = (-b)^2$

$\Rightarrow \sin^2\alpha + \cos^2\alpha + 2\sin\alpha\cos\alpha = (-b)^2$

$\Rightarrow 1 + 2c = b^2\left[\because \sin^2\theta + \cos^2\theta = 1\right]$

$\Rightarrow b^2 - 1 = 2c$

Question3

The general solution of the equation $\sqrt{6 - 5\cos x + 7\sin^2 x} - \cos x = 0$ also satisfies the equation

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Options:

A.

$\tan x + \cot x = 2$

B.

$\cot x + \operatorname{cosec} x = 1$

C.

$\tan x + \sec x = 1$

D.

$\sec x + \operatorname{cosec} x = 2$

Answer: C

Solution:

Given equation

$$\sqrt{6-5\cos x+7\sin^2 x}-\cos x=0$$
$$\Rightarrow\sqrt{6-5\cos x+7-7\cos^2 x}-\cos x=0$$
$$\Rightarrow\sqrt{13-5\cos x-7\cos^2 x}-\cos x=0$$

Squaring both sides, we get

$$\Rightarrow 13-5\cos x-7\cos^2 x=\cos^2 x$$
$$\Rightarrow 13-5\cos x-8\cos^2 x=0$$

$$\Rightarrow 8\cos^2 x+5\cos x+13=0$$

Let $y=\cos x$

$$\Rightarrow 8y^2+5y-13=0$$
$$y=\frac{-5\pm\sqrt{25-4(8)(-13)}}{2\times 8}$$
$$=\frac{-5\pm\sqrt{25+416}}{16}=\frac{-5\pm 21}{16}$$
$$\Rightarrow\quad y=\frac{-5+21}{16}\text{ or } y=\frac{-5-21}{16}$$
$$\Rightarrow\quad y=1\text{ or } y=-\frac{26}{16}=-\frac{13}{8}(not\ possible)$$

$\therefore\quad \cos x=1\Rightarrow\cos x=1$

And $\sin^2 x=1-\cos^2 x=0\Rightarrow\sin x=0$

So, $\tan x=\frac{\sin x}{\cos x}=\frac{0}{1}=0$

$\sec x=\frac{1}{\cos x}=1$

Now, $\tan x+\cot x=0+\frac{1}{0}=\infty$

$$\cot x+\operatorname{cosec} x=\frac{1}{0}+\frac{1}{0}=\infty$$
$$\tan x+\sec x=0+1=1$$
$$\sec x+\operatorname{cosec} x=1+\frac{1}{0}=\infty$$

Question4

Suppose, θ_1 and θ_2 are such that $(\theta_1-\theta_2)$ lies in 3rd or 4th quadrant. If $\sin\theta_1+\sin\theta_2=-\frac{21}{65}$ and $\cos\theta_1+\cos\theta_2=-\frac{27}{65}$, then $\cos\left(\frac{\theta_1-\theta_2}{2}\right)=$

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Options:

A. $\frac{3}{\sqrt{150}}$

B. $\frac{3}{\sqrt{130}}$

C. $-\frac{3}{\sqrt{130}}$

D. $-\frac{3}{\sqrt{150}}$

Answer: C

Solution:

Given the equations:

$\sin\theta_1 + \sin\theta_2 = -\frac{21}{65}$

$\cos\theta_1 + \cos\theta_2 = -\frac{27}{65}$

By squaring both equations and adding them, we have:

$\cos^2\theta_1 + \cos^2\theta_2 + \sin^2\theta_1 + \sin^2\theta_2 + 2\left[\cos(\theta_1 - \theta_2)\right]$

This equates to:

$= \left(\frac{21}{65}\right)^2 + \left(\frac{27}{65}\right)^2$

Calculating further, we get:

$2 + 2\cos(\theta_1 - \theta_2) = \frac{441+729}{65^2}$

Simplifying, we have:

$2\left[2\cos^2\left(\frac{\theta_1-\theta_2}{2}\right)\right] = \frac{1170}{65^2}$

Therefore:

$\cos\left(\frac{\theta_1-\theta_2}{2}\right) = \frac{-3}{\sqrt{130}}$

Given that $\frac{\theta_1-\theta_2}{2}$ lies in the second quadrant, the result confirms cos will be negative in this quadrant. Hence:

$\cos\left(\frac{\theta_1-\theta_2}{2}\right) = -\frac{3}{\sqrt{130}}$

Question5

If A is the solution set of the equation $\cos^2 x = \cos^2\frac{\pi}{6}$ and B is the solution set of the equation $\cos^2 x = \log_{16} P$ where, $P + \frac{16}{P} = 10$,

then, $B - A =$

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Options:

A. $\{x \in R / x = 2n\pi \pm \frac{\pi}{4}, 2n\pi \pm \frac{\pi}{3} n = 0, 12, 3 \ldots\}$

B. $\{x \in R / x = 2n\pi \pm \frac{\pi}{3}, 2n\pi \pm \frac{2\pi}{3} n = 0, 1, 2, 3 \ldots\}$

C. $\{x \in R / x = 2n\pi \pm \frac{\pi}{6}, 2n\pi \pm \frac{\pi}{12} n = 0, 1, 2, 3 \ldots\}$

D. $\{x \in R / x = 2n\pi \pm \frac{\pi}{8}, 2n\pi \pm \frac{\pi}{16} n = 0, 1, 2, 3 \ldots\}$

Answer: B

Solution:

To solve this problem, let's consider the solution sets of the given equations step by step.

Set A:

We start with the equation: $\cos^2 x = \cos^2 \frac{\pi}{6}$.

Since $\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$, we have $\cos^2 \frac{\pi}{6} = \frac{3}{4}$.

The solutions to this equation are:

$$x = n\pi \pm \frac{\pi}{6}, \quad \text{where } n \in \mathbb{Z}$$

Set B:

We consider the equation: $\cos^2 x = \log_{16} P$.

Given $P + \frac{16}{P} = 10$, finding P involves solving the quadratic equation $p^2 - 10p + 16 = 0$.

Factoring gives: $(p-8)(p-2) = 0$, so $p = 8$ or $p = 2$.

Thus, $\cos^2 x = \log_{16} 8$ or $\cos^2 x = \log_{16} 2$.

For $p = 8$, $\log_{16} 8 = \frac{3}{4}$ (since $\frac{3}{4} = \cos^2\left(\frac{\pi}{6}\right)$).

For $p = 2$, $\log_{16} 2 = \frac{1}{4}$.

Hence, $\cos^2 x = \frac{3}{4}$ aligns with $\cos^2 x = \cos^2 \frac{\pi}{6}$.

For $\cos^2 x = \frac{1}{4}$, the solutions are:

$$x = 2n\pi \pm \frac{\pi}{3} \quad \text{and} \quad x = 2n\pi \pm \frac{2\pi}{3}, \quad \text{where } n \in \mathbb{Z}$$

Finding $B - A$:

Set B includes solutions for both $\cos^2 x = \frac{1}{4}$ and $\cos^2 x = \frac{3}{4}$.

Set A only includes solutions for $\cos^2 x = \frac{3}{4}$.

Therefore, the difference $B - A$ consists of solutions only when $\cos^2 x = \frac{1}{4}$, which are:

$B - A = \left\{x \in \mathbb{R} \,|\, x = 2n\pi \pm \frac{\pi}{3},\, 2n\pi \pm \frac{2\pi}{3},\, n \in \mathbb{Z}\right\}$

Question6

If $\tan A + \tan B + \cot A + \cot B = \tan A \tan B - \cot A \cot B$ and $0^\circ < A + B < 270^\circ$, then $A + B =$

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Options:

A. 45°

B. 135°

C. 150°

D. 225°

Answer: B

Solution:

Given the problem:

$\tan A + \tan B + \cot A + \cot B = \tan A \tan B - \cot A \cot B$

We can express the trigonometric functions in terms of sine and cosine. That gives:

$\frac{\sin A}{\cos A} + \frac{\sin B}{\cos B} + \frac{\cos A}{\sin A} + \frac{\cos B}{\sin B} = \frac{\sin A}{\cos A} \cdot \frac{\sin B}{\cos B} - \frac{\cos A}{\sin A} \cdot \frac{\cos B}{\sin B}$

Simplifying each term, we get:

$\frac{\sin^2 A + \cos^2 A}{\sin A \cos A} + \frac{\sin^2 B + \cos^2 B}{\sin B \cos B} = \frac{\sin^2 A \sin^2 B - \cos^2 A \cos^2 B}{\cos A \cos B \sin A \sin B}$

Since $\sin^2 \theta + \cos^2 \theta = 1$, this reduces to:

$\frac{1}{\sin A \cos A} + \frac{1}{\sin B \cos B} = \frac{\sin^2 A \sin^2 B - (1 - \sin^2 A)(1 - \sin^2 B)}{\cos A \cos B \sin A \sin B}$

$\Rightarrow \frac{1}{2}(\sin 2A + \sin 2B) = \sin^2 A + \sin^2 B - 1$

Using the sine addition and subtraction formulas, $\sin(2x) = 2 \sin x \cos x$, transform the equation:

$\sin(A+B)\cos(A-B) = \sin^2 A - \cos^2 B$

Simplifying further:

$\sin(A+B)\cos(A-B) = -\cos(A+B)\cos(A-B)$

This implies:

$\tan(A+B) = -1 \Rightarrow A+B = 135^\circ$

Given the constraint $0^\circ < A+B < 270^\circ$, the valid solution is:

$A+B = 135^\circ$

Question7

The equation that is satisfied by the general solution of the equation $4 - 3\cos^2\theta = 5\sin\theta\cos\theta$ is

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Options:

A. $7\sin^2\theta + 3\cos^2\theta = 4$

B. $\sin^2\theta - 2\cos\theta + \frac{1}{4} = 0$

C. $\cot\theta - \tan\theta = \sec\theta$

D. $1 + \sin^2\theta = 3\cos^2\theta$

Answer: D

Solution:

Given, $4 - 3\cos^2\theta = 5\sin\theta\cos\theta$

$4\sec^2\theta - 3 = 5\tan\theta \quad [\because \text{divide by } \cos^2\theta]$ $4 + 4\tan^2\theta - 3 = 5\tan\theta$

$[\because 1 + \tan^2\theta = \sec^2\theta]$

$4\tan^2\theta - 5\tan\theta + 1 = 0$

$4\tan^2\theta - 4\tan\theta - \tan\theta + 1 = 0$

$(4\tan\theta - 1)(\tan\theta - 1)$

$\tan\theta = \frac{1}{4}, 1$

If $\tan\theta = 1$, then $\theta = 45^\circ$

Now, put the value of θ in option,

$\therefore 1+\sin^2\theta=1+\frac{1}{2}=\frac{3}{2}=3\cos^2\theta$

Hence, $1+\sin^2\theta=3\cos^2\theta$

Question8

The solution set of the equation $\cos^2 2x+\sin^2 3x=1$ i

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Options:

A. $\left\{x \mid x=n\pi+\frac{\pi}{2}, n\in Z\right\}$

B. $\left\{x \mid x=2n\pi\pm\frac{\pi}{4}, n\in Z\right\}$

C. $\left\{x \mid x=\frac{n\pi}{5}, n\in Z\right\}$

D. $\left\{x \mid x=n\pi+(-1)^n\frac{n\pi}{6}, n\in Z\right\}$

Answer: C

Solution:

We have, $\cos^2 2x+\sin^2 3x=1$

$$\begin{aligned}\Rightarrow\quad & 1-\sin^2 2x+\sin^3 3x=1\\ \Rightarrow\quad & \sin^2 3x=\sin^2 2x\Rightarrow 3x=nx\pm 2x\end{aligned}$$

On taking + ve sign, we get

$x=n\pi, n\in Z$

On taking - ve sign, we get

$$\begin{aligned} & 3x=n\pi-2x, n\in Z\\ & 5x=n\pi, n\in Z\\ & x=\frac{n\pi}{5}, n\in Z\\ & x\in\left\{\frac{n\pi}{5} \mid n\in Z\right\}\\ \therefore\quad & x\in\left\{\frac{n\pi}{5} \mid n\in Z\right\}\cup\{n\pi \mid n\in Z\}\\ & =\left\{\frac{n\pi}{5} \mid n\in Z\right\}\end{aligned}$$

Hence, solution set is $\left\{x/x=\frac{n\pi}{5}, n\in Z\right\}$

Question9

If the period of the function $f(x) = 2\cos(3x+4) - 3\tan(2x-3) + 5\sin(5x) - 7$ is k, then

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Options:

A. $\sin \frac{k}{8} = \frac{1}{2}$

B. $\cos \frac{k}{6} = \frac{1}{\sqrt{2}}$

C. $\tan \frac{k}{3} = -\sqrt{3}$

D. $\sec \frac{k}{2} = 2$

Answer: C

Solution:

We have,

$f(x) = 2\cos(3x+4) - 3\tan(2x-3) + 5\sin(5x) - 7$

For $\cos(3x+4)$, period is $\frac{2\pi}{3}$

For $\tan(2x-3)$, period is $\frac{\pi}{2}$

For $\sin(5x)$, period is $\frac{2\pi}{5}$

To find overall period of $f(x)$, we need the LCM of $\frac{2\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{5}$. or LCM of $\frac{40\pi}{60}, \frac{30\pi}{60}, \frac{24\pi}{60}$

So, the overall period k of the function

$f(x)$ is $\dfrac{120\pi}{60} = 2\pi$

$$\Rightarrow \tan \frac{k}{3} = \tan \frac{2\pi}{3} = -\sqrt{3}$$

Question10

The number of solutions of the equation $\sin 7\theta - \sin 3\theta = \sin 4\theta$ that lie in the interval $(0, \pi)$, is

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Options:

A. 6

B. 3

C. 4

D. 5

Answer: D

Solution:

We have,

$\sin 7\theta - \sin 3\theta = \sin 4\theta$

$$2\cos\left(\frac{7\theta + 3\theta}{2}\right)\sin\left(\frac{7\theta - 3\theta}{2}\right) = \sin 4\theta$$

$2\cos 5\theta \sin 2\theta = 2\sin 2\theta \cos 2\theta$

$\sin 2\theta(\cos 5\theta - \cos 2\theta) = 0$

$$\sin 2\theta\left[-\sin\left(\frac{7\theta}{2}\right)\sin\left(\frac{3\theta}{2}\right)\right] = 0$$

$$\sin 2\theta \cdot \sin\frac{7\theta}{2} \cdot \sin\frac{3\theta}{2} = 0$$

$$\sin 2\theta = 0 \Rightarrow \theta = \frac{\pi}{2} \text{ in } (0, \pi)$$

$$\sin\frac{7\theta}{2} = 0 \Rightarrow \theta = \frac{2\pi}{7}, \frac{4\pi}{7}, \frac{6\pi}{7} \text{ is } (0, \pi)$$

$$\sin\frac{3\theta}{2} = 0 \Rightarrow \theta = \frac{2\pi}{3}$$

Hence, total number of solutions of equation $= 5$
